Quiz 8: Brief Review

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1. A recurrence is defined by \( f(n) = 7f(n - 1) + 2f(n - 2) + 11f(n - 3) \)
   with base case \( f(0) = f(1) = f(2) = 1 \). Write the characteristic matrix
   for this recurrence.

2. Consider the string \( S = \text{"ananas"} \), where \( S_0 = \text{"ananas"} \), \( S_1 = \text{"nanas"} \), . . . What are \( S_3 \) and \( \text{head}_3 \)?

3. Using your answer above and the suffix lemma, make the most concrete
   statement about the length of \( \text{head}_4 \) that you can.

4. What is the runtime of using selection sort to sort a list of length \( n \)?

5. What is the worst-case runtime of using merge sort to sort a list of
   length \( n \)?

6. What is the best-case runtime of using any comparison-based sort to
   sort a list of length \( n \)?
7. What is the runtime of naive multiplication between two $n$-digit integers?

8. What is the runtime of Karatsuba multiplication between two $n$-digit integers?

9. What is the runtime of naive matrix multiplication between two $n \times n$ matrices?

10. What is the runtime of Strassen matrix multiplication between two $n \times n$ matrices?

11. A new science-fiction matrix multiplication algorithm has runtime $r(n) = 5 \cdot r \left( \frac{n}{2} \right) + 3n^2$. Use the master theorem to describe the runtime of the algorithm. Specify whether you are using leaf-heavy, root-heavy, or root-leaf balanced. In the case of a root-heavy algorithm, specify the runtime bounds with and without regularity.

12. An algorithm has runtime recurrence $r(n) = 2 \cdot r \left( \frac{n}{2} \right) + n \log(n) \log(n)$. Use the master theorem to describe the runtime of the algorithm. Specify whether you are using leaf-heavy, root-heavy, or root-leaf balanced. In the case of a root-heavy algorithm, specify the runtime bounds with and without regularity.
13. Naive complex multiplication \((a + b \cdot j) \cdot (c + d \cdot j)\) performs how many real multiplications?

14. Gauß complex multiplication \((a + b \cdot j) \cdot (c + d \cdot j)\) performs how many real multiplications?

15. The value of the Fibonacci function grows (circle one):
   - Sub-linearly \((i.e., f(n) \in o(n))\)
   - Linearly \((i.e., f(n) \in \Theta(n))\)
   - Super-linearly but sub-exponentially \((i.e., f(n) \in \omega(n) \text{ and } f(n) \in o(c^n), \text{ for any } c > 1)\)
   - At least exponentially \((i.e., f(n) \in \Omega(c^n), \text{ for some } c > 1)\)

16. Draw the suffix tree for the string \(S = “abcd$”\)

17. Draw the suffix tree for the string \(S = “aaaa$”\)