Quiz 3: Sorting, Recurrence Closed Forms, and Master Theorem

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1. Consider sorting an array of unique values using quicksort (with a random pivot element). After sorting, values $i$ and $j$ end up in indices $r_i$ and $r_j$, where $r_j > r_i$. What is the probability of comparing $i$ and $j$ during sorting?

$$p_{i,j} =$$

2. Construct the characteristic matrix for the following recurrence:

$$z(t) = \begin{cases} 
1 & t = 0 \text{ or } t = 1 \text{ or } t = 2 \\
2z(t-1) + 3z(t-2) - z(t-3) & \text{else.}
\end{cases}$$

3. A divide-and-conquer algorithm has runtime $r(n) = a \cdot r\left(\frac{n}{b}\right) + f(n)$. Name the three cases we consider for the master theorem and the criteria necessary for each case? (You do not need to mention the regularity conditions.)